Appraisal Institute
Empowering your future

Formulas, Symbols, Math Review, and Sample Problems

SMATHREV-E

For Educational Purposes Only

The opinions and statements set forth herein reflect the viewpoint of the Appraisal Institute at the time of publication but do not necessarily reflect the viewpoint of each individual. While a great deal of care has been taken to provide accurate and current information, neither the Appraisal Institute nor its editors and staff assume responsibility for the accuracy of the data contained herein. Further, the general principles and conclusions presented in this text are subject to local, state, and federal laws and regulations, court cases, and any revisions of the same. This publication is sold for educational purposes with the understanding that the publisher and its instructors are not engaged in rendering legal, accounting, or any other professional service.

Nondiscrimination Policy

The Appraisal Institute advocates equal opportunity and nondiscrimination in the appraisal profession and conducts its activities in accordance with applicable federal, state, and local laws.

Mathematics and Analytical Skills Review

I. Order Of Operations

Background—A universal agreement exists regarding the order in which addition, subtraction, multiplication, and division should be performed.

1) Powers and roots should be performed first.

2) Multiplication and division are performed next from left to right in the order that they appear.

3) Additions and subtractions are performed last from left to right in the order that they appear.

Example 1:

$$3 + 4 \times 5 =$$
$$3 + 20 = 23$$

Example 2:

$$7 \times 3^2 =$$
$$7 \times 9 = 63$$

Note. If grouping symbols such as parentheses "( )," brackets "[ ]," and braces "{ }," are present, the operations are simplified by first starting with the innermost grouping symbols and then working outward.

Example 3:

$$\{[12 - 2 \times (7 - 2 \times 2)] \div 3\}^2 =$$
$$\{[12 - 2 \times (7 - 4)] \div 3\}^2 =$$
$$\{[12 - 2 \times 3] \div 3\}^2 =$$
$$\{[12 - 6] \div 3\}^2 =$$
$$\{6 \div 3\}^2 =$$
$$2^2 = 4$$

II. Subtracting/Adding Negative Numbers

Background—Every negative number has its positive counterpart, which is sometimes called its additive inverse. The additive inverse of a number is that number which when added to it produces 0. Thus, the additive inverse of 5 is +5 because (5) + (+5) = 0. Subtracting a negative number is the same as adding its positive counterpart. Adding a negative number is the same as subtracting its positive counterpart.

Example 1:

$$\begin{aligned} 17 - (-3) &= \\ 17 + (3) &= 20 \end{aligned}$$

Example 2:

$$\begin{aligned} 12 + (-8) - (-10) &= \\ 12 + (-8) + (10) &= \\ 12 - (8) + (10) &= 14 \end{aligned}$$

III. Multiplication/Division With Negative Numbers

Background—When numbers of opposite signs are multiplied or divided, the result is negative. When numbers of the same sign are multiplied or divided, the result is always positive. When dividing or multiplying, the two negative signs cancel out.

Example 1:

$$\begin{aligned} 6 \times (-7) \div 3 &= \\ (-42) \div 3 &= (-14) \end{aligned}$$

Example 2:

$$(-32) \div (-4) = 8$$

IV. Addition/Subtraction of Fractions

Background—Simplifying fractions by addition or subtraction requires the use of the lowest common denominator. The denominator on both fractions must be the same before performing an operation. Just as when adding dollars and yen, the yen must be converted to dollars before addition.

Example 1:

$$\$120 + ¥13{,}000 =$$

$$\$120 + ¥13{,}000\left(\frac{\textit{dollars}}{130\ \textit{Yen}}\right) =$$

$$\$120 + \$100 = \$220$$

Example 2:

$$\frac{3}{4} + \frac{2}{3} =$$

$$\left(\frac{3}{3}\right)\frac{3}{4} + \left(\frac{4}{4}\right)\frac{2}{3} =$$

$$\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$$

V. Multiplication/Division of Fractions

Background—Multiplication with fractions is very straightforward, just multiply numerator by numerator and denominator by denominator. When dividing with a fraction, the number being divided (dividend) is multiplied by the reciprocal of the divisor. Frequently this has been stated "invert and multiply."

Example 1:

$$\frac{2}{3} \times \frac{4}{5} = \left(\frac{2 \times 4}{3 \times 5}\right) = \frac{8}{15}$$

Example 2:

$$3 \div \frac{3}{4} = 3 \times \frac{4}{3} = \frac{12}{3} = 4$$

VI. Compound Fractions

Background—Frequently a mathematical expression appears as a fraction with one or more fractions in the numerator and/or the denominator. To simplify the expression multiply the top and bottom of the fraction by the reciprocal of the denominator.

Example:

$$\frac{\frac{2}{5}}{4} =$$

$$\frac{\frac{1}{4} \times \frac{2}{5}}{\frac{1}{4} \times 4} = \frac{\frac{2}{20}}{1} =$$

$$\frac{2}{20} = \frac{1}{10}$$

Note. When multiplying or dividing the numerator and denominator of the fraction by the same number the value of the fraction does not change. In essence, the fraction is being multiplied/divided by 1.

VII. Exponents

Background—Exponents were invented to make it easier to write certain expressions involving repetitive multiplication: $K \times K \times K \times K \times K \times K \times K \times K \times K = K^9$. Note that the exponent (9) specifies the number of times the base (K) is used as a factor rather than the number of times multiplication is performed.

Example: $6^4 = 6 \times 6 \times 6 \times 6 = 1{,}296$

VIII. Fractional Exponents

The definition of a fractional exponent is as follows:

$$X^{M/N} = \sqrt[N]{X^M}$$

This equality converts an expression with a radical sign into an exponent so that the y^x key found on most financial calculators can be used.

Example 1: $12^{4/5} = \sqrt[5]{12^4} = 7.3009$

Example 2: $\sqrt[5]{10} = 10^{1/5} = 10^{0.2} = 1.5849$

IX. Subscripts

Background—Concepts or variables that are used in several equations generally use subscripts to differentiate the values.

Example: Capitalization rates are expressed as a capital "R." Since there are a number of different capitalization rates used by appraisers, a subscript is used to specify which capitalization rate is intended. An equity capitalization rate, therefore, is written as R_E.

X. Percentage Change

Background— Calculating percentage change or delta "Δ" is required in several of the capitalization techniques. The formula for "Δ" is:

$$\Delta = \frac{\text{final value} - \text{starting value}}{\text{starting value}}$$

Example 1: What percentage of change occurs if a property purchased for $90,000 sells for $72,000?

Answer:

$$\Delta = \frac{\$72,000 - \$90,000}{\$90,000} =$$

$$\Delta = \frac{\$18,000}{\$90,000} = -0.20 = -20\%$$

Example 2: What percentage of change occurs if a property purchased for $75,000 sells for $165,000?

Answer:

$$\Delta = \frac{\$165,000 - \$75,000}{\$75,000} =$$

$$\Delta = \frac{\$90,000}{\$75,000} = 120\%$$

XI. Cancellation of Units

Background—Many appraisal applications involve the multiplication and/or division of numbers with “units” associated with them, e.g., $/sf, sf, ft, yr., etc. The proper handling of these units is necessary to describe the mathematical result correctly. According to the identity principle, any number/variable divided by itself is equal to 1 and can thus be removed from the equation.

$$\frac{5}{5} = \frac{X}{X} = \frac{7xy^2}{7xy^2} = 1$$

Example: What value would be indicated for a 12,000 sf building if it is worth $55/sf?

Answer:

$$12{,}000sf \times \frac{\$55}{sf} = \$660{,}000$$

$$sf \times \frac{\$}{sf} = \$$$

XII. Solving Equations

Background—In many instances, an equation or formula exists in a form that is not convenient for the problem at hand, e.g., with value as the goal and the available equation is: I = R × V. Using equation solving techniques, the formula can be rewritten to solve for value with V = I ÷ R as the result.

The rules of equation solving are quite simple and are as follows:

1) Adding or subtracting the same number/variable to both sides of the equation will not change the solution.

2) Multiplying or dividing both sides of the equation by the same number/variable will not change the solution.

3) Raising both sides of the equation by the same power or taking the same root will not change the solution.

Example: *Income = Capitalization Rate × Value*

$I = R \times V$ Divide both sides by R.

$$\frac{I}{R} = \frac{R \times V}{R}$$

$$\frac{I}{R} = V$$

A caution to be noted! Multiplying both sides of an equation by an expression containing a variable or the unknown *could* result in an equation with additional roots that were not in the original equation. This does not change the answer of either equation, nor does it simplify the answer. For example x = 5 has one root, 5. Multiplying both sides by "x" results in $x^2 = 5x$ which has two roots, 0 and 5. Also, dividing both sides of an equation by an expression containing a variable or the unknown could result in an equation losing roots that were in the original equation. For example, $x^2=5x$ has two roots, 5 and 0. Dividing both sides by "x" results in x=5, which has one root 5.

XIII. Problem Solving

Background— Formal problem solving techniques vary from person to person, but usually fall into a sequence of steps that can be categorized as follows:

1) Identify the question to be answered. If the required solution can be represented by a symbol, write it as such. Since many problems require the use of a formula, the identification of information in symbol form helps one recognize potential formula(s) that might be used to solve the problem.

2) Carefully glean all of the given data from the problem statement and assign symbols, if possible. The data may be represented as a number, a word, or a phrase ($10,000, six, value will double during the projection period).

3) Based on the identification and assignment of symbols in steps 1 and 2, attempt to list (mentally or on paper) all of the methods or techniques (frequently a formula) that you are aware of that can be used to find the answer.

4) Compare the list in step 3 with the data from steps 1 and 2. Based on this comparison, one of the following situations will emerge:

    a) The solution is fairly obvious, and all of the necessary information has already been identified.

    b) The solution is fairly obvious, but some additional data must be created from the given information.

    c) The solution is not obvious, and the items in the list in step 3 must be considered one-by-one until a correct one is found.

    d) “a,” “b,” and “c” fail to solve the problem. Steps 2 and 3 may have been improperly handled and must be revisited with “a,” “b,” and “c” retried. In the worst case scenario (not in an Appraisal Institute course), the problem may not be solvable.

Example: What is the present value of $1,500 per year for 12 years discounted at 15%?

Step 1: Solve for present value or PV.

Step 2: Identify all variables:

Years	=	12
Discount rate	=	15%
Annuity	=	$1,500

Step 3: Possible equations:

$$PV = CF\left[\frac{1-\left(\frac{1}{(1+i)^n}\right)}{i}\right]$$

Step 4: With only one possible equation and all the variables accounted for, this problem becomes straightforward.

$$PV = \$1,500\left[\frac{1-\left(\frac{1}{(1+0.15)^{12}}\right)}{0.15}\right]$$

$$= \$1,500\left[\frac{1-\left(\frac{1}{5.35025}\right)}{0.15}\right]$$

$$= \$1,500\left[\frac{0.813093}{0.15}\right]$$

= $1,500 [5.4206]

= $8,130.93

Summary of Basic Formulas

❶ Direct Capitalization

Where:		
I	=	Income
R	=	Capitalization Rate
I_O	=	Net Operating Income
V	=	Value
M	=	Mortgage Ratio
DCR	=	Debt Coverage Ratio (also called Debt Service Coverage Ratio)
F	=	Capitalization Factor (Multiplier)
GIM	=	Gross Income Multiplier
EGIM	=	Effective Gross Income Multiplier
NIR	=	Net Income Ratio

Subscripts:		
O	=	Overall Property
M	=	Mortgage
E	=	Equity
L	=	Land
B	=	Building

Basic Income/Cap Rate/Value Formulas

$$\frac{\mathbf{I}}{\mathbf{R \times V}}$$

$$I = R \times V$$
$$R = I/V$$
$$V = I/R$$

Basic Value/Income/Factor Formulas

$$\frac{\mathbf{V}}{\mathbf{I \times F}}$$

$$V = I \times F$$
$$I = V/F$$
$$F = V/I$$

Cap Rate/Factor Relationships

$$R = 1/F$$
$$R_O = NIR/GIM$$
$$R_O = NIR/EGIM$$

Note. NIR may relate to Scheduled Gross or Effective Gross Income, and care should be taken to ensure consistency.

Adaptations for Mortgage/Equity Components

Band of Investment (using ratios):

$R_O = M \times R_M + [(1 - M) \times R_E]$

$R_E = (R_O - M \times R_M)/(1 - M)$

Equity Residual:

$V_O = [(I_O - V_M \times R_M)/R_E] + V_M$

$R_E = (I_O - V_M \times R_M)/V_E$

$R_E = R_O + (R_O - R_M) \times [M/(1 - M)]$

Mortgage Residual:

$V_O = [(I_O - V_E \times R_E)/R_M] + V_E$

$R_M = (I_O - V_E \times R_E)/V_M$

Debt Coverage Ratio:

$DCR = I_O/I_M$

$R_O = DCR \times M \times R_M$

$DCR = R_O/(M \times R_M)$

$M = R_O/(DCR \times R_M)$

$V_M = I_O/(DCR \times R_M)$

Adaptations for Land/Building Components

Land and building band-of-investment formula:

Where:

L = land to total value ratio

B = building to total value ratio

$R_O = (L \times R_L) + (B \times R_B)$

Land Residual:

$V_O = [(I_O - V_B \times R_B)/R_L] + V_B$

$R_L = (I_O - V_B \times R_B)/V_L$

Building Residual:

$V_O = [(I_O - V_L \times R_L)/R_B] + V_L$

$R_B = (I_O - V_L \times R_L)/V_B$

❷ Yield Capitalization

Where:		
PV	=	Present Value
CF	=	Cash Flow
Y	=	Yield Rate
R	=	Capitalization Rate
Δ	=	Change
a	=	Annualizer
$1/S_{\overline{n}\rceil}$	=	Sinking Fund Factor
1/n	=	1/Projection Period
CR	=	Compound Rate of Change
V	=	Value

Subscripts:		
n	=	Projection Period
O	=	Overall Property
I	=	Income

Discounted Cash Flows/Present Value (DCF/PV)

$$PV = \frac{CF_1}{(1+Y)^1} + \frac{CF_2}{(1+Y)^2} + \frac{CF_3}{(1+Y)^3} + \ldots + \frac{CF_n}{(1+Y)^n}$$

Basic Cap Rate/Yield Rate/Value Change Formulas

$$R = Y - \Delta a$$

$$Y = R + \Delta a$$

$$\Delta a = Y - R$$

$$\Delta = (Y - R)/a$$

Adaptations for Common Income/Value Patterns

Pattern	Premise	Cap Rates (R)	Yield Rates (Y)	Value Changes (Δ)
Perpetuity	$\Delta = 0$	$R = Y$	$Y = R$	
Level Annuity*	$a = 1/S_{\overline{n}\rceil}$	$R = Y - \Delta 1/S_{\overline{n}\rceil}$	$Y = R + \Delta 1/S_{\overline{n}\rceil}$	$\Delta = (Y - R)/(1/S_{\overline{n}\rceil})$
St. Line Change	$a = 1/n$	$R = Y - \Delta 1/n$	$Y = R + \Delta 1/n$	$\Delta = (Y - R)/(1/n)$
Exponential Change	$\Delta_O a = CR$	$R_O = Y_O - CR$	$Y_O = R_O + CR$	$\Delta_O = (1 + CR)^n - 1$

* Inwood Premise: $1/S_{\overline{n}\rceil}$ at Y rate; Hoskold Premise: $1/S_{\overline{n}\rceil}$ at safe rate

St. Line Change* in Income	St. Line Change* in Value	Compound Rate of Change
$\$\Delta_I = V \times \Delta 1/n \times Y$	$\$\Delta 1/n = \$\Delta_I/Y$	$CR = \sqrt[n]{FV/PV} - 1$
$\Delta_I = (Y \times \Delta 1/n)/(Y - 1/n)$	$\Delta 1/n = (Y \times \Delta_I)/(Y + \Delta_I)$	$CR = Y_O - R_O$

* Δ_I in these formulas is the ratio of one year's change in income related to the first year's income.

❸ Present Value of Increasing/Decreasing Annuities

Straight Line Changes

To obtain the present value of an annuity that has a starting income of **d** at the end of the first period and *increases h dollars* per period for **n** periods:

$$PV = (d + hn)\, a_{\overline{n}\rceil} - [h\,(n - a_{\overline{n}\rceil})]/i$$

To obtain the present value of an annuity that has a starting income of **d** at the end of the first period and *decreases h dollars* per period for **n** periods, simply treat **h** as a negative quantity in the foregoing formula.

Constant Ratio (Exponential Curve) Changes

To obtain the present value of an annuity that starts at $1 at the end of the first period and *increases each period* thereafter at the rate **x** for **n** periods:

$$PV = [1 - (1 + x)^n/(1 + i)^n]/(i - x)$$

Where: **i** is the periodic discount rate and **x** is the ratio of the increase in income for any period to the income for the previous period.

To obtain the present value of an annuity that starts at $1 at the end of the first period and *decreases each period* thereafter at rate **x**, simply treat rate **x** as a negative quantity in the foregoing formula.

❹ Mortgage-Equity Analysis

Where:		
r	=	Basic Capitalization Rate
Y	=	Yield Rate
M	=	Mortgage Ratio
P	=	Ratio Paid Off—Mortgage
$1/S_{\overline{n}\rceil}$	=	Sinking Fund Factor
R	=	Capitalization Rate
$S_{\overline{n}\rceil}$	=	Future Value of $1 Per Period
Δ	=	Change
J	=	J-Factor (Changing Income)
n	=	Projection Period
I_O	=	Net Operating Income
B	=	Mortgage Balance
I	=	Nominal Interest Rate
T	=	Effective Tax Rate

Subscripts:		
E	=	Equity
M	=	Mortgage
P	=	Projection
O	=	Overall Property
I	=	Income
ET	=	After-tax Equity
OT	=	After-tax Property Yield
1	=	1st Mortgage
2	=	2nd Mortgage

Leverage Relationships

Pre-tax Equity Capitalization Rates

If $R_O > R_M$, then $R_E > R_O$ and leverage is positive

If $R_O = R_M$, then $R_E = R_O$ and leverage is neutral

If $R_O < R_M$, then $R_E < R_O$ and leverage is negative

Using Pre-tax Equity Yield Rates

If $Y_O > Y_M$, then $Y_E > Y_O$ and leverage is positive

If $Y_O = Y_M$, then $Y_E = Y_O$ and leverage is neutral

If $Y_O < Y_M$, then $Y_E < Y_O$ and leverage is negative

After-tax Yield Rates (where Y_{OT} is the after-tax property yield. Y_{ET} is the after-tax equity yield and T is the effective tax rate.

If $Y_{OT} > Y_M (1 - T)$, then $Y_{ET} > Y_{OT}$ and leverage is positive

If $Y_{OT} = Y_M (1 - T)$, then $Y_{ET} = Y_{OT}$ and leverage is neutral

If $Y_{OT} < Y_M (1 - T)$, then $Y_{ET} < Y_{OT}$ and leverage is negative

---

Mortgage/Equity Formulas

BASIC CAPITALIZATION RATES (R)

$$r = Y_E - M(Y_E + P\ 1/S_{n\rceil} - R_M)$$

$$r = Y_E - M_1(Y_E + P_1\ 1/S_{n\rceil} - R_{M1}) - M_2(Y_E + P_2\ 1/S_{n\rceil} - R_{M2})$$

$$P = (R_M - I)/(R_{MP} - I)$$

$$P = \frac{1/S_{n\rceil}}{1/S_{n\rceil P}}$$

CAPITALIZATION RATES (R)

Level Income:

$$R_O = Y_E - M(Y_E + P\ 1/S_{n\rceil} - R_M) - \Delta_O 1/S_{n\rceil}$$

$$R_O = r - \Delta_O 1/S_{n\rceil}$$

J-Factor Changing Income:

$$R_O = [Y_E - M(Y_E + P\ 1/S_{n\rceil} - R_M) - \Delta_O 1/S_{n\rceil}]/(1 + \Delta_I J)$$

$$R_O = (r - \Delta_O 1/S_{n\rceil})/(1 + \Delta_I J)$$

K-Factor Changing Income:

$$R_O = [Y_E - M\ (Y_E + P\ 1/S_{n\rceil} - R_M) - \Delta_O 1/S_{n\rceil}]/K$$

Required Change in Value (Δ):

Level Income:

$\Delta_O = (r - R)/(1/S_{\overline{n}|})$

$\Delta_O = [Y_E - M\,(Y_E + P\,1/S_{\overline{n}|} - R_M) - R]/1/S_{\overline{n}|}$

J-Factor Changing Income:

$\Delta_O = [r - R_O\,(1 + \Delta_I J)]/(1/S_{\overline{n}|})$

$*\Delta_O = (r - R_O)/(R_O J + 1/S_{\overline{n}|})$

Note. For multiple mortgage situations, insert $M\,(Y_E + P\,1/S_{\overline{n}|} - R_M)$ for each mortgage.
* This formula assumes value and income change at the same ratio.

EQUITY YIELD (Y_E)

Level Income:

$Y_E = R_E + \Delta_E\,1/S_{\overline{n}|}$

J-Factor Changing Income:

$Y_E = R_E + \Delta_E\,1/S_{\overline{n}|} + [R_O\,\Delta_I/(1 - M)]\,J$

K-Factor Changing Income:

$Y_E = R_E + \Delta_E\,1/S_{\overline{n}|} + [R_O\,(K - 1)/(1 - M)]$

CHANGE IN EQUITY

$\Delta_E = (\Delta_O + MP)/(1 - M)$

or

$\Delta_E = [V_O\,(1 + \Delta_O) - B - V_E]/V_E$

Ellwood Without Algebra Format

1.	Mortgage ratio × annual constant	=	Weighted rate
2.	Equity ratio × equity yield rate	=	Weighted rate
			Weighted average
Less credit for equity buildup:			
3.	Mortgage ratio × portion paid off × *SFF*	=	(credit)
			Basic rate, *r*
Adjustment for *dep* or *app*:			
4.	Plus *dep* (or minus *app*) × *SFF*	=	adjustment
			Cap rate, *R*

Note. *SFF* is the sinking fund factor for equity yield rate and projection period. *Dep* or *app* is the fraction of value lost from depreciation or gained from appreciation during the projection period.

The capitalization rate resulting from this calculation is only applicable to level income situations. Adjustment is necessary for application to non-level income patterns.

❺ Investment Analysis

Where:		
PV	=	Present Value
NPV	=	Net Present Value
CF	=	Cash Flow
i	=	Discount Rate in NPV Formula
n	=	Projection Period
IRR	=	Internal Rate of Return
PI		Profitability Index
MIRR	=	Modified Internal Rate of Return
$FVCF_j$	=	Future Value of a Series of Cash Flows
i	=	Reinvestment Rate in MIRR Formula

Subscripts:		
0	=	At Time Zero
1	=	End of 1st Period
2	=	End of 2nd Period
3	=	End of 3rd Period
n	=	End of Last Period in Series

Net Present Value (NPV)

$$NPV = CF_0 + \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \ldots + \frac{CF_n}{(1+i)^n}$$

Internal Rate of Return (IRR)

Where: NPV = 0; IRR = i

Profitability Index (PI)

$PI = PV/CF_0$

Modified Internal Rate of Return (MIRR)

$$\text{MIRR} = \sqrt[n]{FVCF_i / CF_0} - 1$$

$$\text{MIRR} = \sqrt[n]{[CF_1 (1 + i)^{n-1} + CF_2 (1 + i)^{n-2} + CF_3 (1 + i)^{n-3} + \ldots + CF_n]/CF_0} - 1$$

Note. In these formulas individual CFs may be positive or negative for PV and NPV solutions. However, CF_0 is treated as a positive value for PI and MIRR solutions.

Nominal Rate vs Real Rates

(1 + nominal rate) = (1 + real rate) (1 + expected inflation rate)

Effective Tax Rate

T = (Pretax Y – Aftertax Y)/Pretax Y

Variance

$$\sum_{i=1}^{n} P_i (x_i - \overline{x})^2$$

Standard Deviation

$$\sqrt{\text{variance}}$$

Expected Return is found by

$$\overline{x} = \sum_{i=1}^{n} P_i X_i$$

Breakeven Occupancy

$$\frac{\text{Expenses} + I_M}{\text{Annual rent per unit}}$$

Breakeven Ratio

$$\frac{\text{Expenses} + I_M}{\text{Gross Income}}$$

❻ Symbols

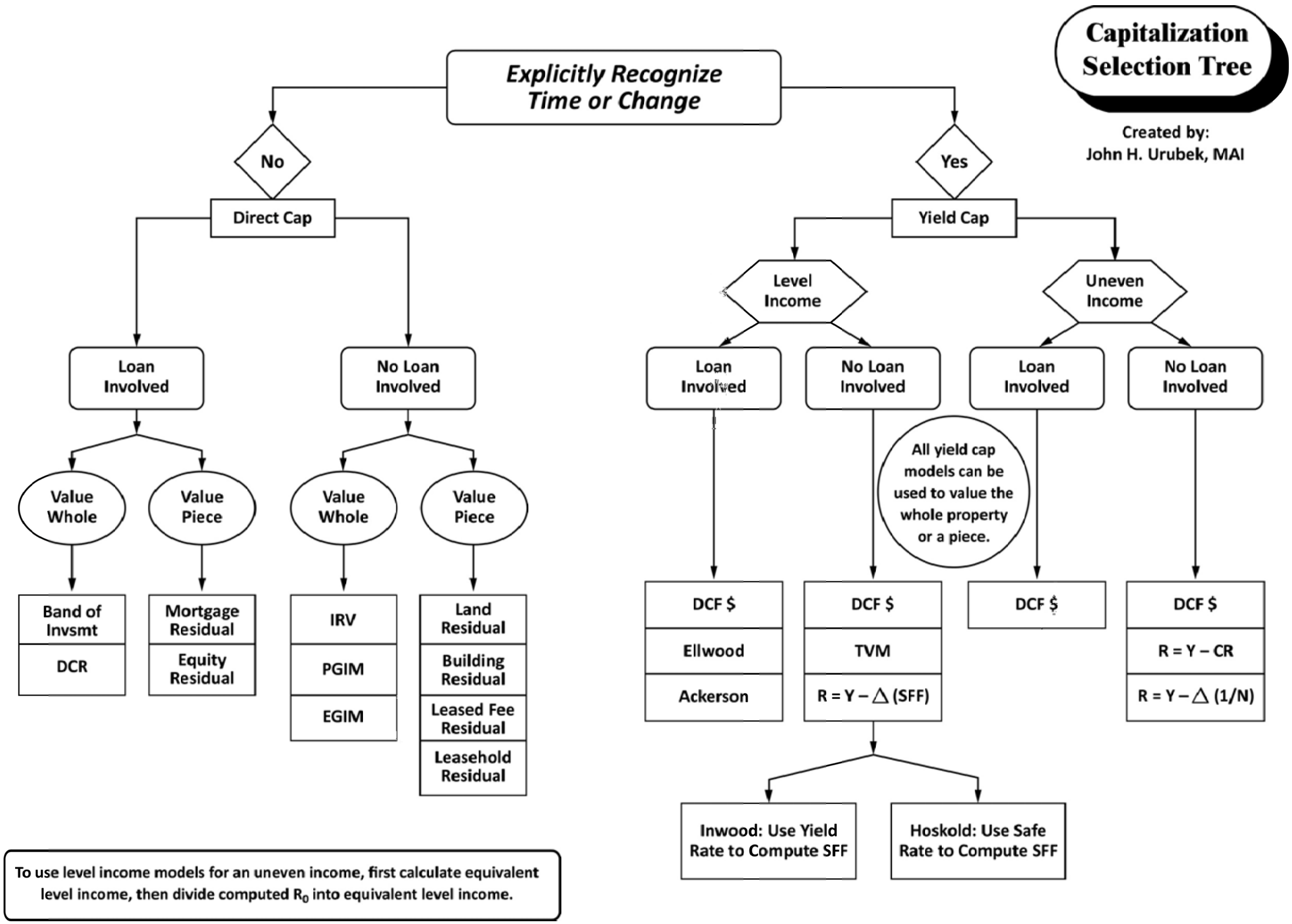
Symbol	Description
$1/S_{n\rceil}$	Sinking fund factor
$S_{n\rceil}$	Future value of an amount per period
$1/S^n$	Present value factor
$1/a_{n\rceil}$	Partial payment factor
S^n	Future value of an amount
$a_{n\rceil}$	Present value of an amount per period
$\$\Delta$	Total change in dollars
Δ	Total percent change (expressed as a decimal)
Δ_E	Total percent change in equity (expressed as a decimal)
Δ_I	Total percent change in income (expressed as a decimal)
Δ_O	Total percent change in property value (expressed as a decimal)
a	Annualizer to convert Δ to an annual change
A	Adjustment rate to reflect the change in value ($A = \Delta a$)
ATCF	After-tax cash flow
B	Building-to-property value ratio
B	Remaining balance on a mortgage expressed as a percentage of original mortgage amount.
CF	Cash flow. Sometimes followed by a numerical subscript that indicates period number.
CO	Cash outlays
CR	Compound rate of change
DCR	Debt coverage ratio
EGI	Effective gross income
FV	Future value
i	Interest rate
I	Income

Symbol	Description
I_B	Income to the building
I_E	Income to the equity
I_L	Income to the land
I_{LF}	Income to the leased fee
I_{LH}	Income to the leasehold
I_M	Income to the mortgage, same as the debt service
I_O	Net Operating Income (NOI)
IRR	Internal rate of return. A yield rate
J	J factor
K	K factor used for constant-ratio changes in income
L	Land-to-property value ratio
L_n	Natural Logarithm
M	Loan-to-value ratio
MIRR	Modified internal rate of return
n	Number of periods
NOI	Net Operating Income. Sometimes the symbol I_O is used.
NPV	Net Present Value
P	Percent of mortgage paid off, usually at the end of the study period
PGI	Potential gross income
PMT	Periodic payment
PTCF	Pre-tax cash flow to equity (I_E)
PV	Present Value
r	"Basic capitalization rate"
R	Capitalization rate
R_B	Building capitalization rate
R_E	Equity capitalization rate
R_L	Land capitalization rate
R_M	Mortgage capitalization rate or annual constant

Symbol	Description
R_N	Terminal capitalization rate
R_o	Overall capitalization rate. Property capitalization rate
T	Effective tax rate
V	Value
V_B	Building value
V_E	Equity value
V_L	Land value
V_{LF}	Leased fee value
V_{LH}	Leasehold value
V_M	Mortgage value
V_{NOI}	Value of the income stream to the property
V_O	Property value
V_{REV}	Present value of the reversion (also shown as V_{PR})
Y	Yield rate
Y_E	Equity yield rate
Y_{ET}	After-tax equity yield rate
Y_{LF}	Yield to the leased fee
Y_{LH}	Yield to the leasehold
Y_M	Mortgage yield rate
Y_{MT}	After tax yield rate to mortgage
Y_O	Property yield rate
Y_{OT}	After-tax property yield rate

❼ Standard Subscripts

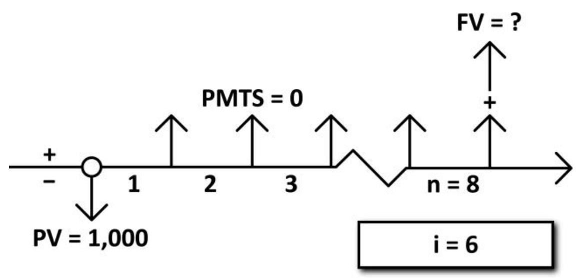
Subscript	Description
B	Building
E	Equity
I	Income
L	Land
LF	Leased fee
LH	Leasehold
M	Mortgage
O	Overall property
N	Terminal period
REV	Reversion
PR	Property reversion


Capitalization Selection Tree
Created by:
John H. Urubek, MAI
Explicitly Recognize Time or Change
No
Yes
Direct Cap
Yield Cap
Loan Involved
No Loan Involved
Value Whole
Value Piece
Band of Invsmt
DCR
Mortgage Residual
Equity Residual
IRV
PGIM
EGIM
Land Residual
Building Residual
Leased Fee Residual
Leasehold Residual
Level Income
Uneven Income
Loan Involved
No Loan Involved
Loan Involved
No Loan Involved
All yield cap models can be used to value the whole property or a piece.
DCF $
Ellwood
Ackerson
DCF $
TVM
R = Y – △ (SFF)
DCF $
DCF $
R = Y – CR
R = Y – △ (1/N)
Inwood: Use Yield Rate to Compute SFF
Hoskold: Use Safe Rate to Compute SFF
To use level income models for an uneven income, first calculate equivalent level income, then divide computed R0 into equivalent level income.

Sample Problems with Suggested Solution Keystrokes for the HP-10B, HP-12C, HP-17B, and HP-19B*

1. Future Value of $1.00

If $1,000 is deposited in an account earning 6.0 percent per year, what will the account balance be at the end of 8 years?

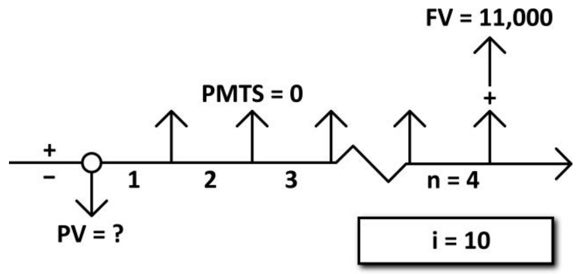


Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	8 N	8 n	8 N
5	Enter interest rate.	6 I/YR	6 i	6 I%YR
6	Enter beginning balance.	1000 +/− PV	1000 CHS PV	1000 +/− PV
7	Ensure cleared payment register.	0 PMT	0 PMT	0 PMT
8	Calculate future balance.	FV	FV	FV
The account balance will be $1,593.85.				

* Set HP-12C Platinum, HP-17B, and HP-19B calculators to RPN mode.

2. Present Value of $1.00

What is the present value of the right to receive $11,000 in four years at a discount rate of 10.0 percent per year?

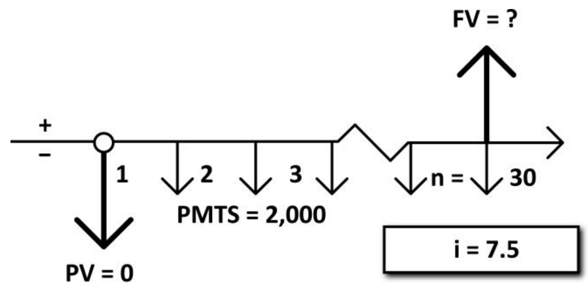


Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	4 N	4 n	4 N
5	Enter interest rate.	10 I/YR	10 i	10 I%YR
6	Ensure cleared payment register.	0 PMT	0 PMT	0 PMT
7	Enter future value.	11000 FV	11000 FV	11000 FV
8	Calculate present value.	PV	PV	PV

The present value is $7,513.15. (The display of –7,513.15 reflects the sign convention of the calculator.) **Note.** The cash flows are presented from the perspective of the investor purchasing the right to receive the future income.

3. Future Value of $1.00 Per Period

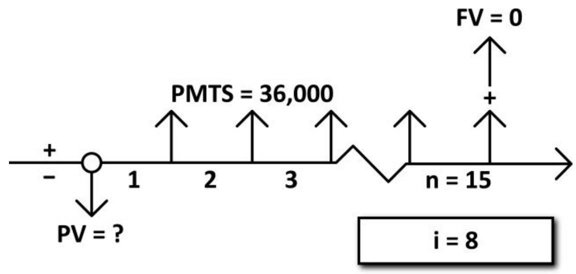
What will be the value of an Individual Retirement Account in 30 years assuming that deposits of $2,000 are made at the end of each year and the account earns 7.5 percent per year?



Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	30 N	30 n	30 N
5	Enter interest rate.	7.5 I/YR	7.5 i	7.5 I%YR
6	Enter payment amount.	2000 +/– PMT	2000 CHS PMT	2000 +/– PMT
7	Ensure cleared present value register.	0 PV	0 PV	0 PV
8	Calculate future value.	FV	FV	FV
The account value will be $206,798.81.				

4. Present Value of $1.00 Per Period (Annual Cash Flows)

What is the present value of the right to receive a payment of $36,000 at the end of every year for 15 years at a discount rate of 8.0 percent?

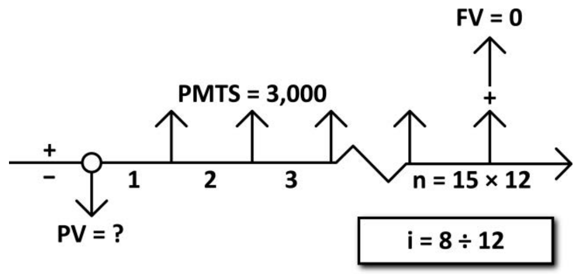


Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	15 N	15 n	15 N
5	Enter interest rate.	8 I/YR	8 i	8 I%YR
6	Enter payment amount.	36000 PMT	36000 PMT	36000 PMT
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate present value.	PV	PV	PV

The present value is $308,141.23. (The display of –308,141.23 reflects the sign convention of the calculator.) **Note.** The cash flows are presented from the perspective of the investor purchasing the right to receive the future cash flows.

5. Present Value of $1.00 Per Period (Monthly Cash Flows)

What is the present value of the right to receive a payment of $3,000 at the end of every month for 15 years at a discount rate of 8.0 percent?

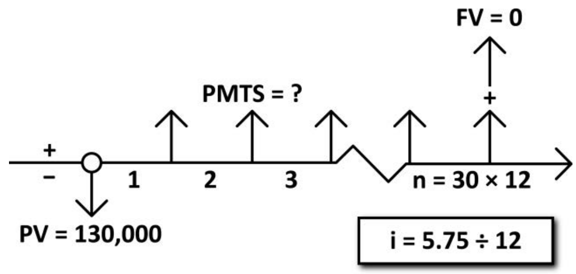


Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter number of periods.	15 gold xP/YR	15 g n	15 gold N
5	Enter interest rate.	8 I/YR	8 g i	8 I%YR
6	Enter payment amount.	3000 PMT	3000 PMT	3000 PMT
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate present value.	PV	PV	PV

The present value is $313,921.78. (The display of –313,921.78 reflects the sign convention of the calculator.) **Note.** The cash flows are presented from the perspective of the investor purchasing the right to receive the future cash flows.

6. Partial Payment Factor (Installment to Amortize $1.00)

What monthly payment is necessary to fully amortize a $130,000 loan in 30 years at an interest rate of 5.75 percent per year?

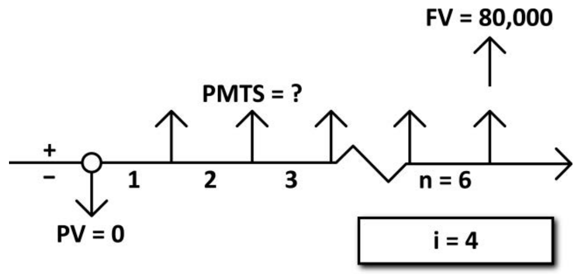


Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter number of periods.	30 gold xP/YR	30 g n	30 gold N
5	Enter interest rate.	5.75 I/YR	5.75 g i	5.75 I%YR
6	Enter loan amount.	130000 +/− PV	130000 CHS PV	130000 +/− PV
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate payment.	PMT	PMT	PMT

The monthly payment is $758.64. **Note.** The cash flows presented are from the perspective of the lender.

7. Sinking Fund Factor

How much must be deposited at the end of each year into an account that earns 4.0 percent interest to have an account balance of $80,000 at the end of six years?



Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	6 N	6 n	6 N
5	Enter interest rate.	4 I/YR	4 i	4 I%YR
6	Enter future value.	80000 FV	80000 FV	80000 FV
7	Ensure cleared present value register.	0 PV	0 PV	0 PV
8	Calculate required deposit amount (payment).	PMT	PMT	PMT

The annual payment is $12,060.95. (The display of –12,060.95 reflects the sign convention of the calculator.) **Note.** The cash flows are presented from the perspective of the investor establishing the sinking fund.

8. Calculating a Loan Balance

What will be the balance at the end of the tenth year on a monthly payment $130,000 loan with a 30-year amortization period at an interest rate of 5.75 percent per year?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter number of periods.	30 gold xP/YR	30 g n	30 gold N
5	Enter interest rate.	5.75 I/YR	5.75 g i	5.75 I%YR
6	Enter loan amount.	130000 +/− PV	130000 CHS PV	130000 +/− PV
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate payment.	PMT	PMT	PMT
The monthly payment is $758.64.				
9	Change holding period.	10 gold N	10 g n	10 gold N
10	Calculate future value.	FV	FV	FV
The future value (loan balance) is $108,056.19.				

9. Loan Term

How long will it take to pay off a loan which has a current balance of $58,000 and an interest rate of 7.5 percent per year if the monthly payments are $850.00?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter interest rate.	7.5 I/YR	7.5 g i	7.5 I%YR
5	Enter current loan amount.	58000 +/− PV	58000 CHS PV	58000 +/− PV
6	Enter monthly payment.	850 PMT	850 PMT	850 PMT
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate number of periods.	N	n	N
It will take 90 months to pay off the loan. (The HP-12C reports 90 months, meaning that 90 payments will be required; the HP-10B, 17B, and 19B report 89.23 months, indicating that it will take longer than 89 months to pay the loan off, but that the final payment will be smaller than $850.00.)				

10. Mortgage Yield with Points

What will be the lender's yield on a monthly payment $130,000 loan with a 30-year amortization period and an interest rate of 5.75 percent per year if the lender charges the buyer a loan fee of three points?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter number of periods.	30 gold xP/YR	30 g n	30 gold N
5	Enter interest rate.	5.75 I/YR	5.75 g i	5.75 I%YR
6	Enter loan amount.	130000 +/− PV	130000 CHS PV	130000 +/− PV
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate payment.	PMT	PMT	PMT
The monthly payment is $758.64.				
9	Recall present value.	RCL PV	RCL PV	RCL PV
10	Deduct points.	− 3% =	3% −	3% −
11	Store new value in present value.	PV	PV	PV
12	Calculate periodic yield rate.	I/YR	i	I%YR
13	Calculate annual yield rate.	N/A	12 x	N/A
The lender's yield rate is 6.03 percent. **Note.** The suggested keystrokes are based on having the 17B or 19B calculator set to RPN, not algebraic.				

11. Cash Equivalent Value of a Loan

What is the cash equivalent value of a monthly payment $130,000 loan provided by the seller of a property if it has a 30-year amortization period and an interest rate of 5.75 percent per year, and the market interest rate is 7.0 percent?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	12 gold P/YR	N/A	OTHER 12 P/YR EXIT
4	Enter number of periods.	30 gold xP/YR	30 g n	30 gold N
5	Enter contract interest rate.	5.75 I/YR	5.75 g i	5.75 I%YR
6	Enter loan amount.	130000 +/− PV	130000 CHS PV	130000 +/− PV
7	Ensure cleared future value register.	0 FV	0 FV	0 FV
8	Calculate payment.	PMT	PMT	PMT
The monthly payment is $758.64.				
9	Enter market interest rate.	7 I/YR	7 g i	7 I%/YR
10	Calculate present value.	PV	PV	PV
The cash equivalent value of the loan is $114,030.04. (The display of −114,030.04 reflects the sign convention of the calculator.) Note. The cash flows are presented from the perspective of the lender.				

12. Leased Fee Valuation (Level Income)

A property is subject to a lease with level payments of $32,500 per year and there are five years remaining on the lease. At the end of the lease term, the property is expected to be sold for a net price of $450,000. What is the value of the leased fee interest in the property at a yield rate of 13%?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	5 N	5 n	5 N
5	Enter yield rate.	13 I/YR	13 i	13 I%YR
6	Enter payment.	32500 PMT	32500 PMT	32500 PMT
7	Enter future value.	450000 FV	450000 FV	450000 FV
8	Calculate present value.	PV	PV	PV
The present value is $358,551.99. (The display of –358,551.99 reflects the sign convention of the calculator.) **Note.** The cash flows are presented from the perspective of the investor purchasing the right to receive the future cash flows and reversion.				

13. Leased Fee Valuation (Non-Level Income)

A property is subject to a lease with a remaining term of five years. The first-year rent is $30,000, and the rent will increase $2,000 per year. At the end of the lease term, the property is expected to be sold for a net price of $450,000. What is the value of the leased fee interest in the property at a yield rate of 13%?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select CFLO menu.	N/A	N/A	FIN CFLO
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Clear the cash flow list.	gold C ALL	f REG	gold CLEAR DATA YES
5	Enter the cash flow for period 0.	0 CFj	N/A	0 INPUT
6	Enter the cash flow for period 1.	30000 CFj	30000 g CFj	30000 INPUT INPUT
7	Enter the cash flow for period 2.	32000 CFj	32000 g CFj	32000 INPUT INPUT
8	Enter the cash flow for period 3.	34000 CFj	34000 g CFj	34000 INPUT INPUT
9	Enter the cash flow for period 4.	36000 CFj	36000 g CFj	36000 INPUT INPUT
10	Add the total cash flow for period 5 (the rent plus the reversion).	488000 CFj	488000 g CFj	488000 INPUT INPUT
11	Enter yield rate.	13 I/YR	13 i	EXIT CALC 13 I%
12	Calculate present value.	gold NPV	f NPV	NPV
The present value is $362,119.39.				

14. Net Present Value

What is the net present value if the property described in the previous question can be purchased for $350,000? (The property is subject to a lease with a remaining term of five years. The first-year rent is $30,000, and the rent will increase $2,000 per year. At the end of the lease term, the property is expected to be sold for a net price of $450,000. The required yield rate is 13%.)

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select CFLO menu.	N/A	N/A	FIN CFLO
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Clear the cash flow list.	gold C ALL	f REG	gold CLEAR DATA YES
5	Enter the cash flow for period 0.	350000 +/– CFj	350000 CHS g CFo	350000 +/– INPUT
6	Enter the cash flow for period 1.	30000 CFj	30000 g CFj	30000 INPUT INPUT
7	Enter the cash flow for period 2.	32000 CFj	32000 g CFj	32000 INPUT INPUT
8	Enter the cash flow for period 3.	34000 CFj	34000 g CFj	34000 INPUT INPUT
9	Enter the cash flow for period 4.	36000 CFj	36000 g CFj	36000 INPUT INPUT
10	Add the total cash flow for period 5 (the rent plus the reversion).	488000 CFj	488000 g CFj	488000 INPUT INPUT
11	Enter yield rate.	13 I/YR	13 i	EXIT CALC 13 I%
12	Calculate present value.	gold NPV	f NPV	NPV
The net present value is $12,119.39.				

15. Internal Rate of Return (Level Income)

What is the internal rate of return on a property purchased for $250,000 if the annual cash flow is $20,000 and the property is resold at the end of five years for $320,000?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select TVM menu.	N/A	N/A	FIN TVM
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Enter number of periods.	5 N	5 n	5 N
5	Enter purchase price.	250000 +/− PV	250000 CHS PV	250000 +/− PV
6	Enter payment.	20000 PMT	20000 PMT	20000 PMT
7	Enter future value.	320000 FV	320000 FV	320000 FV
8	Calculate internal rate of return.	I/YR	i	I%YR
The internal rate of return is 12.37 percent.				

16. Internal Rate of Return (Non-Level Income)

What is the internal rate of return on a property purchased for $250,000 if the first-year cash flow is $20,000, the income rises by 4.0 percent per year, and the property is resold at the end of five years for $320,000?

Suggested Solution				
Step	**Explanation**	**HP-10B**	**HP-12C**	**HP-17B/ HP-19B**
1	Move to top menu.	N/A	N/A	gold MAIN
2	Select CFLO menu.	N/A	N/A	FIN CFLO
3	Enter number of payments per year.	1 gold P/YR	N/A	OTHER 1 P/YR EXIT
4	Clear the cash flow list.	gold C ALL	f REG	gold CLEAR DATA YES
5	Enter the cash flow for period 0.	250000 +/− CFj	250000 CHS g CFo	250000 +/− INPUT
6	Enter the cash flow for period 1.	20000 CFj	20000 g CFj	20000 INPUT INPUT
7	Enter the cash flow for period 2.	20800 CFj	20800 g CFj	20800 INPUT INPUT
8	Enter the cash flow for period 3.	21632 CFj	21632 g CFj	21632 INPUT INPUT
9	Enter the cash flow for period 4.	22497 CFj	22497 g CFj	22497 INPUT INPUT
10	Enter the total cash flow for period 5 (the income plus the reversion).	343397 CFj	343397 g CFj	343397 INPUT INPUT
11	Calculate yield rate.	gold IRR/YR	f IRR	EXIT CALC IRR%
The internal rate of return is 12.91 percent.				